CIRCULAR CONTACT OF NONLINEAR ELASTIC BODIES SUBJECTED TO IMPORTANT STRAINS

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Abstract: This paper presents some experimental investigations upon circular contacts between bodies subjected to large strains. The circular contact between nonlinear elastic bodies, modelled by a rubber ball pressed against a sapphire window, was investigated by aid of laser profilometry and optical microscopy. The correlation between normal approach and the radial extension of contact area was studied in the case of large strains. For validation, obtained results are compared against ones found in literature.

1. INTRODUCTION

One of the defining characteristics of a hertz contact is that the bodies in contact suffer very small strain in comparison to their macroscopic dimensions [3], [4], [5]. When polymeric materials are involved, even at relatively small loads, the strains are very large reported to the body dimensions. For example, in the case of a synthetic rubber ball pressed against a sapphire window, the ratio r/R was found to range from 0,1 to 0,7, for applied loads of 10 to 50N, for the ball radius in the range $R = 3 - 5mm$, r is the contact area radius, [5].

Occurrence of large strains in contact can have an important influence on the final body geometry. Figure 1 illustrates the geometry of a spherical punch before and after suffering large deformations.

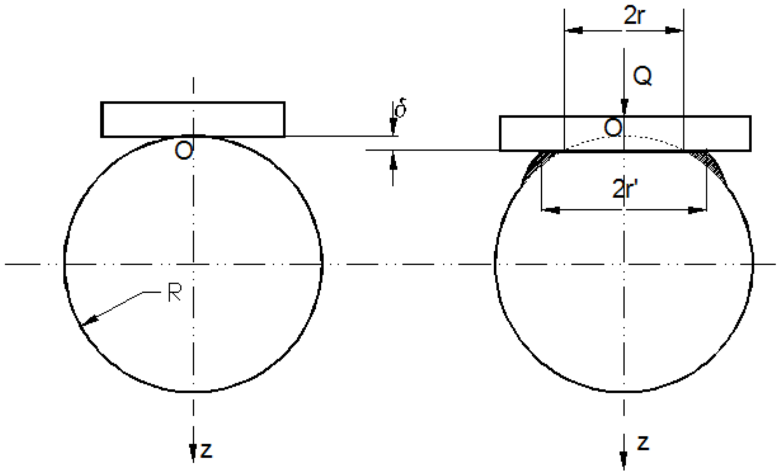


Figure 1. Elastic bodies in contact, undeformed and subjected to important strains

It can be noticed that, as shown in Figure 1, in this situation the contact area is much larger than the one resulting from the application of Hertz formulae. For simplification, the spherical punch was modeled as deformable and the flat surface was considered rigid. Also, the deformable body was considered as nonlinear elastic, incompressible, homogenous and isotropic. The contact of the two bodies is considered frictionless.

2. EXPERIMENTAL SET-UP AND METHODS

In order to conduct the proposed experimental investigations, an experimental device was built as illustrated in Figure 2.

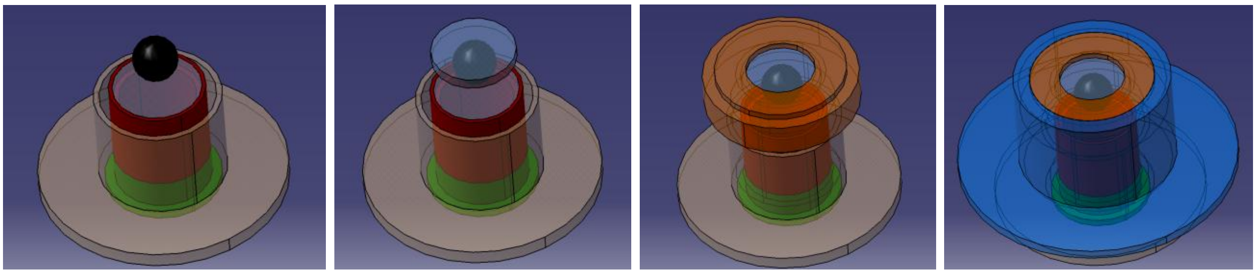


Figure 2. Experimental device for the investigation of elastic bodies in contact

The apparatus shown in Figure 2 allows loading of the contact between a punch and a flat sapphire window by aid of circular dead weights placed on the exterior bush. After contact loading, the flat sapphire surface will descend towards the base with a quantity equal to 2δ, where δ represents the normal approach between the two bodies in contact.

Experimental investigations were conducted for the contact of a synthetic rubber ball with a diameter of 12,6 mm, pressed against a 2 mm thick sapphire window.

The shape and dimensions of the contact area were determined using an optical microscope fitted with a camera. In order to quantitatively evaluate contact area dimensions, a body of known dimensions was placed under the sapphire window. This allows accurate determination of contact area dimensions from an image of the contact, by comparing contact area to the additional body known size.

The normal approach of the two bodies in contact was accurately determined using a laser profilometer. The laser beam is continuously focused on the upper sapphire surface, and the profile height is registered. After contact loading, the sapphire window suffers a vertical displacement. The relative difference between measured profile heights before and after contact loading provides an accurate measurement of twice the normal approach.

3. EXPERIMENTAL RESULTS

Circular contacts between nonlinear elastic bodies undergoing important strains were investigated using the experimental set-up presented above. Typical results obtained for contact elements are illustrated in this paper.

Figure 3 illustrates the experimentally measured normal approach plotted against applied load.

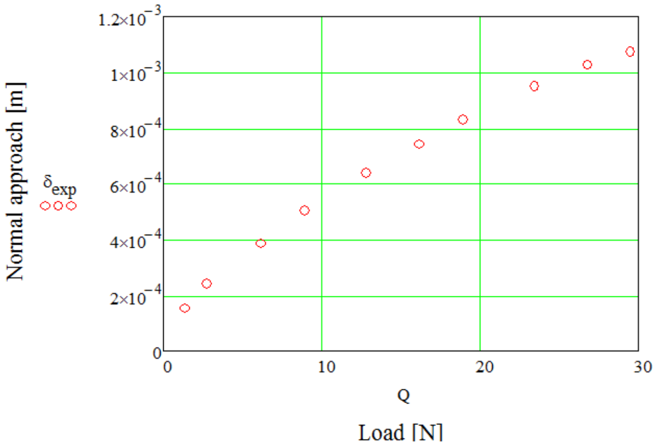
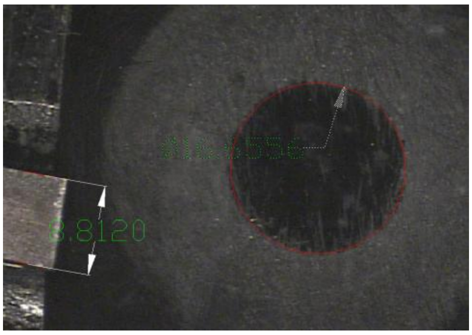
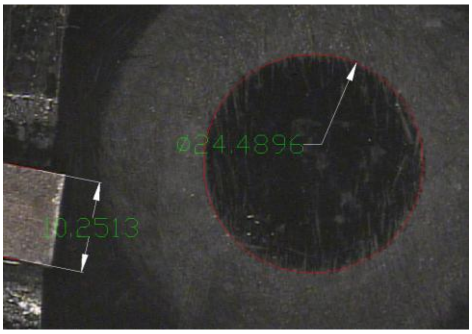


Figure 3. Normal approach variation with load level,
for the contact between a rubber ball and a sapphire window

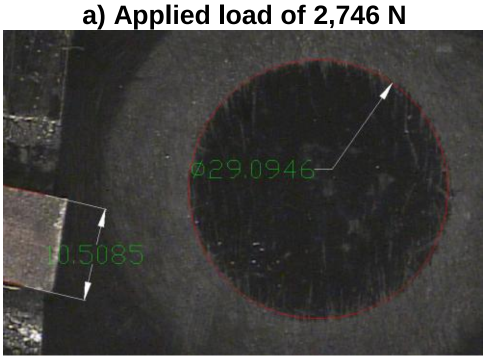
Using dedicated software to process contact area images obtained by optical microscopy, contact area diameters were measured at different load levels. Typical results are shown in Figure 4.



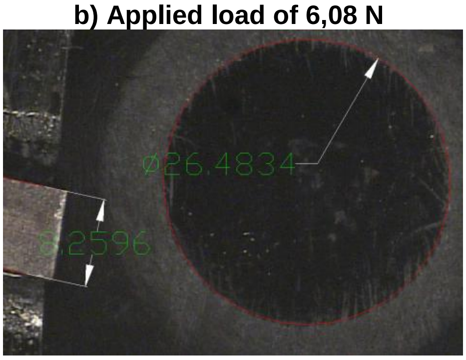
a) Applied load of 2,746 N



b) Applied load of 6,08 N

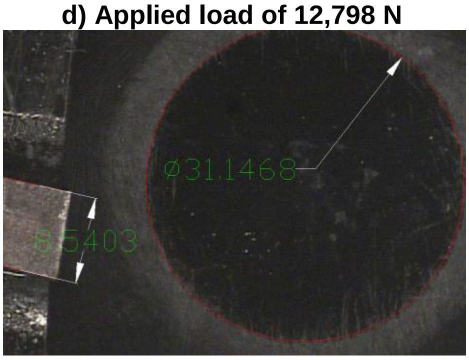


c) Applied load of 8,875 N

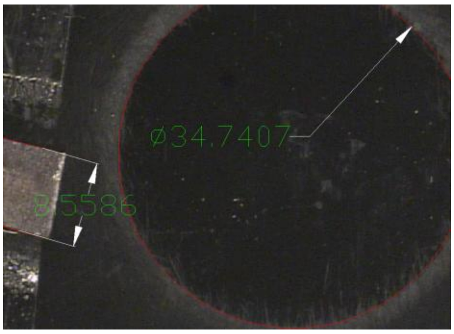


d) Applied load of 12,798 N

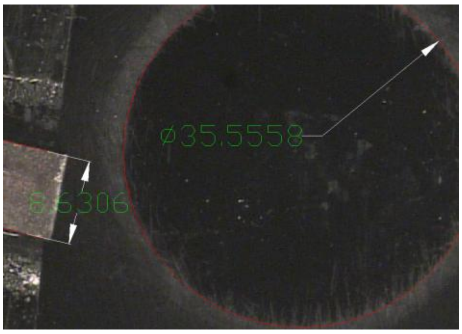
e) Applied load of 16,132 N



f) Applied load of 18,927 N



g) Applied load of 23,389 N



h) Applied load of 26,723 N

Figure 4. Contact area images for a rubber ball ($R = 12,6mm$ ***), pressed against a flat sapphire window***

Contact area diameters indicated in Figure 4 a) – h), must be translated into real values. The conversion is accomplished using a scale factor, obtained as ratio of the indicated dimension, to the known, real, dimension of the additional body. The additional body used in the presented investigations was 1,34 mm wide.

Table 1. Real values of contact area diameters

Load [N]	Aditional body indicated dimension	Indicated contact area dimension	Scale factor	Real value of contact area diameter [mm]
1,32	7,80	12,18	0,17	2,09
2,75	8,81	16,66	0,15	2,53
6,08	10,25	24,49	0,13	3,20
8,88	10,51	29,09	0,13	3,71
12,80	8,26	26,48	0,16	4,30
16,13	8,58	29,48	0,16	4,60
18,93	8,54	31,15	0,16	4,89
23,39	8,56	34,74	0,16	5,44
26,72	8,63	35,56	0,16	5,52

The variation of contact area dimensions with applied normal loads was plotted in Figure 5.

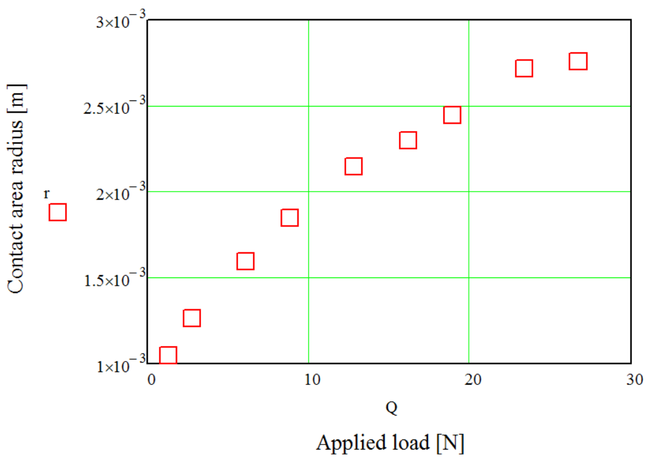


Figure 5. Contact area radius variation with applied load

4. VALIDATION OF EXPERIMENTAL RESULTS

Experimental values measured for the normal approach were plotted against applied load in Figure 6. For comparison, theoretical values, obtained by application of Hertz formulae for linear elastic bodies in contact, were traced on the same diagram.

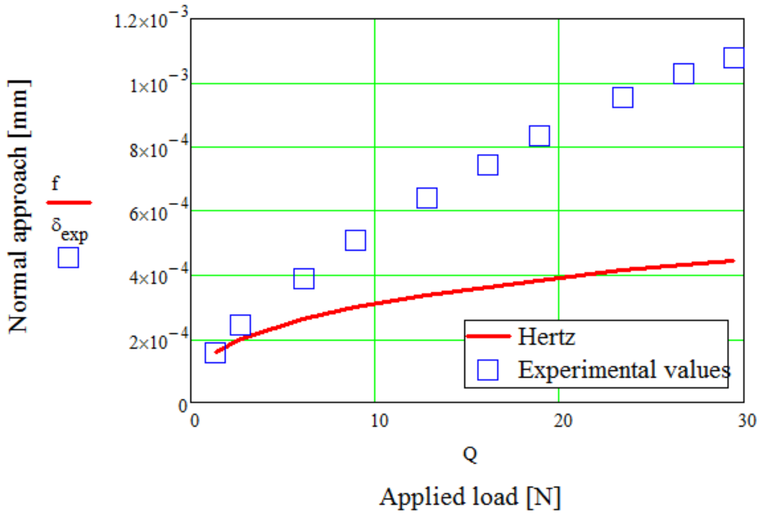


Figure 6. Normal approach to load dependency
(Hertz results correspond to $E_0 = 3,1MPa$***)***

As shown in figure 6, for low load levels, the measured normal approach was found similar to the computed for linear elastic bodies in contact, and the lateral extension can be considered negligible. As the load increases however, significant differences appear. This kind of normal approach to load evolution was deduced by Diaconescu, in [2], for circular contacts between nonlinear elastic bodies, and by Tatara, in [6], [7], for nonlinear bodies subjected to large deformations.

Using the experimentally measured normal approach between bodies, and considering that the ball is compressible, contact area radii, r, were determined from the geometrical deformation condition:

$$r = \frac{1}{2}\sqrt{8\cdot\delta\cdot\left(R-\frac{\delta}{2}\right)}, \tag{4.1}$$

where, δ is the normal approach and R represents ball radius.

The difference between calculated contact area radius and the one measured experimentally represents the radial expansion of the contact area.

The evolution of contact area dimensions with load was then plotted using both experimental and computed values, in the case of incompressible / perfectly compressible bodies in contact.

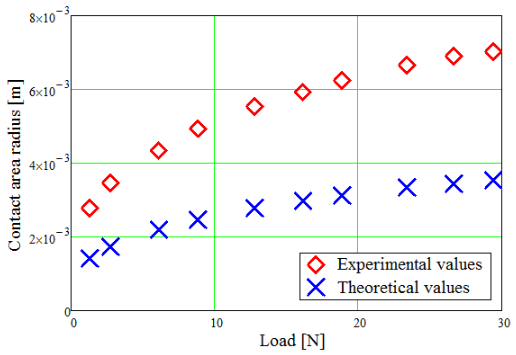
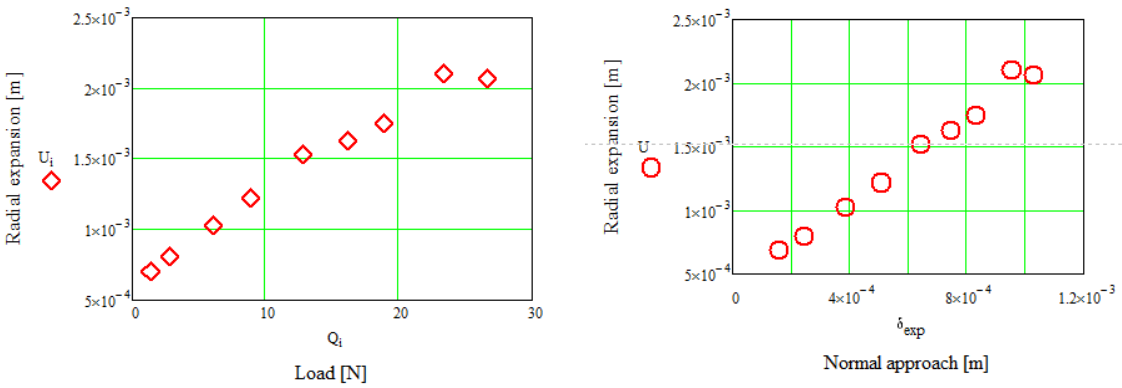


Figure 7. Contact area dimensions vs load

As illustrated in figure 7, if the bodies in contact were compressible, then the resulting areas would be smaller. Strains occurring in rubber bodies are important, and lead to displacements in a radial direction. It can be noticed from figure 7, that for low load levels, contact area dimensions in the two situations are very similar, and as load increases, the two plots became more and more apart.

The evolution of contact area extension was plotted against normal load and normal approach respectively.



a) b)

Figure 8. Radial expansion of the contact area evolution: a) with load; b) with normal approach

As illustrated by figure 8, it was found that contact area radial expansion is very small, even negligible at low loads, while at higher loads it must betaken into consideration, as it becomes significant.

5. CONCLUSIONS

The work presented herein can be summarized by the following conclusions:

- Hertz formulae no longer apply for circular contacts between nonlinear elastic bodies subjected to important strain. This resulted from experimental investigations conducted upon contact area dimensions in the case of a 12,6mm rubber ball pressed against a flat sapphire window.
- When bodies in contact suffer large strains, the resulting contact area suffers a radial expansion. The size of the radial extension increases with normal approach.
- Normal approach between nonlinear elastic bodies is more significant than in the case of linear elastic bodies in contact.
- The lateral extension of the contact area can no longer be neglected if the normal approach to ball radius ratio exceeds 0,05.

6. ACKNOWLEDGEMENT

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